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A MODIFIED SRIF FILTER USING LDL' FACTORIZATIONS.(U)

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A Modified SRIF Filter Using LDL′ Factorizations

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May 15, 1978

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The square-root information filter (SRIF filter), which is a numerical method of implementing the Kalman filter, was investigated. The SRIF filter was modified to accomodate LDL'factorization of the covariance rather than the usual LL'factorizations. This modification led to eliminating the square-root operations in the prewhiting process and a simple means of implementing a fading memory.

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CONTENTS

A MODIFIED SRIF FILTER USING LDL′ FACTORIZATIONS

INTRODUCTION

The problem of estimating a set of parameters or variables from a set of measurements has long been of interest. Kalman in the early sixties provided a simple recursive estimation procedures by introducing the concept of state and state transition. This procedure in some instances provided simpler implementation than batching techniques. Since Kalman's work a number of numerical procedures have been developed. An excellent account of these procedures as well as historical notes can be found in Bierman's book [1]. Two basic techniques described in Ref. 1 are the square-root information filter (SRIF filter) and the UDU filter (where UDU should actually be UDU′, with U being an upper triangular matrix and D being a diagonal matrix). In both cases the data are prewhitened using Cholesky factorization. The SRIF filter is then based on the Householder transform. The UDU filter is based on a Cholesky factorization of the smoothed covariance update using one measurement at a time and the modified Gram Schmidt for updating the predicted covariance. These numerical techniques are claimed to have better numerical stability than direct use of the Kalman filter equations and may be more amenable to hardware implementation.

In Ref. 1 the SRIF filter described used a Cholesky factorization of the covariance matrix of the form of a triangular matrix times the transpose of the same triangular matrix. This report is concerned with reformulating the SRIF filter in terms of a Cholesky factorization of the covariance matrix using a triangular matrix times a diagonal matrix times the transpose of the same triangular matrix, where the diagonal elements of the triangular matrix are 1. The reason for changing the factorization is to attempt to remove some of the square-root operations.

The following section briefly reviews the Kalman filter. The SRIF filter is next derived using the alternate factorization. The properties of a modified Householder algorithm which are necessary for the SRIF-filter mechanization are shown. The prediction process is shown for the case of no process noise, and an exponential time weighting into the past is incorporated. Finally the entire algorithm is written out and examined.

MODIFIED SRIF FILTER

The SRIF filter is a numerical method of implementing the Kalman filter [1]. The Kalman filter is obtained from modeling the process as state equations, defining a measurement procedure, and best-estimating the states of the systems. The state equation and measurement process are defined as

$$X(k) = \Phi(k)\ X(k-1) + \Gamma(k)\ W(k)$$

and

$$X_M(k) = H(k)\ X(k) + V(k),$$

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where it is desired to best-estimate the n-by-1 state vector $X(k)$. The remaining quantities are n-by-n state transition matrix $\Phi(k)$, an n-by-p matrix $\Gamma(k)$, an m-by-n measurement matrix $H(k)$, and an m-by-1 measurement vector $X_M(k)$, with $W(k)$ and $V(k)$ being Gaussian noises with the following properties:

$$E\Big[W(k)\Big] = 0,$$

$$E\Big[W(k)\ W'(j)\Big] = S(k)\ \delta_{jk},$$

$$E\Big[V(k)\Big] = 0,$$

$$E\Big[V(k)\ \nu(j)\Big] = Q(k)\ \delta_{jk},$$

and

$$E\Big[W(k)\ V(j)\Big] = 0,$$

in which

$\delta_{jk} = 1$ when $j = k$ and 0 otherwise.

The covariance matrices $S(k)$ and $Q(k)$ are of dimension p by p and m by m, respectively.

The best estimate of $X(k)$, denoted by $\tilde{X}(k)$ in the standard Kalman-filter format, is

$$\tilde{X}(k) = \hat{X}(k) + K(k)\left[X_M(k) - H(k)\ \hat{X}(k)\right], \tag{1}$$

where $K(k)$ is the filter gain, given by

$$K(k) = \tilde{P}(k)\ H'(k)\ Q^{-1}(k). \tag{2}$$

In equation (2) $\tilde{P}(k)$ is the smoothed covariance matrix, given by

$$\tilde{P}^{-1}(k) = \hat{P}^{-1}(k) + H'(k)\ Q^{-1}(k)\ H(k). \tag{3}$$

In equation (3) $\hat{P}(k)$ is the predicted covariance matrix, with

$$\hat{P}(k+1) = \Phi(k+1)\ \tilde{P}(k)\ \Phi'(k+1) + \Gamma(k+1)$$
$$S(k+1)\ \Gamma'(k+1). \tag{4}$$

The prediction is

$$\hat{X}(k+1) = \Phi(k+1)\ \tilde{X}(k). \tag{5}$$

The filter operates in a predict-and-correct fashion. This suggests the following simple derivation.

Equation (1) is the least-square estimate between the prediction and the measurement at the kth sample, which is obtained by minimizing the cost function

$$J(k) = \left[\hat{X}(k) - X(k)\right]'\ \hat{P}^{-1}(k)\left[\hat{X}(k) - X(k)\right]$$
$$+ \left[X_M(k) - HX(k)\right]' Q^{-1}(k)\ \left[X_m(k) - HX(k)\right] \tag{6}$$

with respect to $X(k)$. The value of $X(k)$ which minimizes $X(k)$, denoted by $\tilde{X}(k)$, is the best estimate of $X(k)$ and is given in equations (1) through (3). When the best estimate of $\tilde{X}(k)$

is given, the best prediction is simply equation (5) with the covariance of (4). The process is then simply repeated recursively with equations (4) and (5) being the prediction and equations (1) through (3) being the correction.

The SRIF filter is a means of implementing the Kalman filter which depends heavily on Cholsky decomposition and the Householder matrix triangulation algorithm [1]. The Cholsky decomposition is performed on a symmetric positive-definite matrix by factoring it to the product of a lower triangular matrix L and its transpose:

$$Q = L\ L'$$

and

$$Q^{-1} = (L')^{-1}\ L^{-1}.$$

The algorithm for obtaining L, found in Ref. 1, is

$$\ell_{jj} = \sqrt{q_{jj}}, \quad j = 1,\ n-1,$$

$$\ell_{kj} = q_{kj}/\ell_{jj}, \quad k = j+1,\ \ldots,\ n,$$

and

$$q_{ik} = q_{ik} - \ell_{ij}\ell_{kj}, \quad k = j+1, \ldots,\ n \text{ and } i = k,\ \ldots,\ n.$$

In this report the LDL' factorization described in Ref. 1 is used to construct the SRIF-like filter rather than using the $L\ L'$ factorization. The algorithm for obtaining the LDL' factorization is

$$Q = L\ D\ L'$$

and

$$Q^{-1} = (L')^{-1}\ D^{-1}\ L^{-1},$$

where

$$\ell_{jj} = 1, \quad j = 1,\ \ldots,\ n-1,$$

$$d_j = q_{jj},$$

$$\ell_{kj} = q_{kj}/d_j, \quad k = j+1,\ \ldots,\ n,$$

and

$$q_{ik} = q_{ik} - \ell_{ij}\ell_{kj}d_j, \quad k = j+1,\ \ldots,\ n \text{ and } i = k,\ \ldots,\ n.$$

The matrix L is lower triangular in form and has diagonal elements of 1. The matrix D is a diagonal matrix with diagonal elements of d_j.

The derivation of a SRIF filter using the LDL' factorization proceeds as follows. The cost function in equation (6) can be written as

$$J = (\hat{X} - X)'\ \hat{R}\ \hat{D}\ \hat{R}'(\hat{X} - X) + (X_M - HX)'$$
$$(L')^{-1}\ D_M^{-1}\ L^{-1}(X_M - HX), \tag{7}$$

where the k denoting sample number has been dropped for notational convenience, $\hat{P}^{-1}(k)$ is factored into $\hat{R}\ \hat{D}\ \hat{R}'$, and $Q(k)$ is factored into $L\ D_M\ L'$ (so that $Q^{-1}(k) = (L')^{-1}\ D_M^{-1}\ L^{-1}$). Equation (7) can be rewritten as

$$J = (\hat{Z} - \hat{R}'X)'\ \hat{D}(\hat{Z} - \hat{R}'Z) + (Z_M - H_W X)'\ D_M^{-1}(Z_M - H_W X), \tag{8}$$

where

$$\hat{Z} = \hat{R}'X,$$

$$Z_M = L^{-1}\ X_M,$$

and

$$H_W = L^{-1}H.$$

Equation (8) can be rewritten more compactly as

$$J = \left[\begin{bmatrix}\hat{R}'\\ H_W\end{bmatrix} X - \begin{bmatrix}\hat{Z}\\ Z_M\end{bmatrix}\right] \begin{bmatrix}\hat{D} & 0\\ 0 & D_M^{-1}\end{bmatrix} \left[\begin{bmatrix}R'\\ H_W\end{bmatrix} X - \begin{bmatrix}\hat{Z}\\ Z_M\end{bmatrix}\right] \tag{9}$$

With the notation

$$D = \begin{bmatrix}\hat{D} & 0\\ 0 & D_M^{-1}\end{bmatrix},$$

the cost function is unaltered if an orthogonal transform T, where $T'D\ T = D$, is multiplied by the new resulting vector in equation (9). Consquently use of

$$\begin{bmatrix}\hat{R}'\\ H_W\end{bmatrix} X - \begin{bmatrix}\hat{Z}\\ Z_M\end{bmatrix} = C$$

in $J = C'\ D\ C$ yields the same cost as

$$J = C'T'D\ T\ C.$$

In addition, if T, which is a n-plus-m square matrix, is chosen such that

$$T\begin{bmatrix}\hat{R}'\\ H_W\end{bmatrix} = \begin{bmatrix}\bar{R}'\\ 0\end{bmatrix} \tag{10}$$

and

$$T\begin{bmatrix}\hat{Z}\\ Z_M\end{bmatrix} = \begin{bmatrix}\bar{Z}\\ e\end{bmatrix}, \tag{11}$$

the cost then becomes

$$J = (\bar{R}'X - \bar{Z})'\ \hat{D}(\bar{R}'X - \bar{Z}) + e'D_M^{-1}\ e. \tag{12}$$

By inspection the least-square estimate of X is

$$\bar{R}'\tilde{X} = \bar{Z}, \text{ or } \tilde{X} = (\bar{R}')^{-1}\ \bar{Z}, \tag{13}$$

the minimum value of the cost J is $e'D_M^{-1}e$, and the smoothed covariance is $\tilde{P}(k) = \bar{R}'D\ \bar{R}$. For simplification equation (10) is augmented to equation (11) yielding

$$T\begin{bmatrix} \hat{R}' & \hat{Z} \\ H_W & Z_M \end{bmatrix} = \begin{bmatrix} \bar{R}' & \bar{Z} \\ 0 & e \end{bmatrix} \tag{14}$$

Later it will be shown that the smoothed covariance factor $\bar{R}$ does not have all 1's on the diagonal elements as the factors $\hat{R}$ and L did. This can be modified by relating the factors $\bar{R}$ and $\hat{D}$ to new factors $\tilde{R}$ and $\tilde{D}$, where $\tilde{R}$ has diagonal elements of 1. This can be shown by rewriting the cost of equation (12) as

$$J = (X - \tilde{X})'\ \bar{R}\ \hat{D}\ \bar{R}'\ (X - \tilde{X}) + e'D_M^{-1}\ e.$$

Since the smoothed covariance can be rewritten as $\tilde{P} = \bar{R}\ \hat{D}\ \bar{R}'$ or $\tilde{P} = \tilde{R}\ \tilde{D}\ \tilde{R}$, the cost function can be rewritten with either way unaltered. The relation can be found by noting the 2-by-2 example

$$\begin{bmatrix} \bar{r}_{11} & 0 \\ \bar{r}_{21} & \bar{r}_{22} \end{bmatrix} \begin{bmatrix} \hat{d}_1 & 0 \\ 0 & \hat{d}_2 \end{bmatrix} \begin{bmatrix} \bar{r}_{11} & \bar{r}_{21} \\ 0 & \bar{r}_{22} \end{bmatrix}$$

which can be rewritten as

$$\begin{bmatrix} 1 & 0 \\ \bar{r}_{21}/\bar{r}_{11} & 1 \end{bmatrix} \begin{bmatrix} \bar{r}_{11} & 0 \\ 0 & \bar{r}_{22} \end{bmatrix} \begin{bmatrix} \hat{d}_1 & 0 \\ 0 & \hat{d}_2 \end{bmatrix} \begin{bmatrix} \bar{r}_{11} & 0 \\ 0 & \bar{r}_{22} \end{bmatrix} \begin{bmatrix} 1 & \bar{r}_{21}/\bar{r}_{11} \\ 0 & 1 \end{bmatrix} \tag{15}$$

Combining terms in equation (15) yields

$$\begin{bmatrix} 1 & 0 \\ \bar{r}_{21}/\bar{r}_{11} & 1 \end{bmatrix} \begin{bmatrix} \hat{d}_1\bar{r}_{11}^2 & 0 \\ 0 & \hat{d}_2\bar{r}_{22}^2 \end{bmatrix} \begin{bmatrix} 1 & \bar{r}_{21}/\bar{r}_{11} \\ 0 & 1 \end{bmatrix}$$

This is the desired factored form $\tilde{R}\ \tilde{D}\ \tilde{R}'$. The algorithm can easily be generalized to yield

$$\tilde{d}_i = \hat{d}_i\ \bar{r}_{ii}^2,\quad i = 1,\ \dots,\ n, \tag{16}$$

and

$$\tilde{r}_{ji} = \bar{r}_{ji}/\bar{r}_{ii},\quad j = i,\ \dots,\ n. \tag{17}$$

The least-square estimate of $\tilde{X}$ in equation (13) is given by $\bar{R}'\tilde{X} = \bar{Z}$, which is also altered into the form

$$\tilde{R}'\tilde{X} = \tilde{Z}. \tag{18}$$

By example again, equation (13) can be written as

$$\begin{bmatrix} \bar{r}_{11} & \bar{r}_{12} \\ 0 & \bar{r}_{22} \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} = \begin{bmatrix} \bar{z}_1 \\ \bar{z}_2 \end{bmatrix},$$

which can be written as

$$\begin{bmatrix} \bar{r}_{11} & 0 \\ 0 & \bar{r}_{22} \end{bmatrix} \begin{bmatrix} 1 & \bar{r}_{21}/\bar{r}_{11} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} = \begin{bmatrix} \bar{z}_1 \\ \bar{z}_2 \end{bmatrix},$$

which in turn can be rewritten as

$$\begin{bmatrix} 1 & \bar{r}_{21}/\bar{r}_{11} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} = \begin{bmatrix} z_1/\bar{r}_{11} \\ z_2/\bar{r}_{22} \end{bmatrix}. \tag{19}$$

Equation (19) is the desired form (18), where

$$\tilde{z}_i = \bar{z}_i/\bar{r}_{ii}, \quad i = 1, \ldots, n. \tag{20}$$

As a summary of the preceding results, the transform T is applied to the augmented matrix equation (14) and the desired factored form is obtained by applying equation (16), (17), and (20). We next consider how the matrix transform T which triangulates a matrix with the property $T\ D'T = D$ is obtained.

MODIFIED HOUSEHOLDER TRANSFORM

The elementary Householder transform given in Ref. 1 is obtained as follows. Let the vector U be normal to the plane $U_\perp$. An arbitrary vector Y can be represented by

$$Y = (Y'\hat{U})\ \hat{U} + V, \tag{21}$$

where $\hat{U} = U/\sqrt{U'U}$ and V is that part of Y that is orthogonal to U. The reflection of Y, denoted by Y_r, in the plane $U_\perp$ is

$$Y_r = -(Y'\hat{U})\hat{U} + V \tag{22}$$

and is represented in Fig. 1. Eliminating V from equations (21) and (22) yields

$$Y_r = Y - 2\frac{(Y'U)}{(U'U)}\ U = (I - \beta UU')\ Y = TY, \tag{23}$$

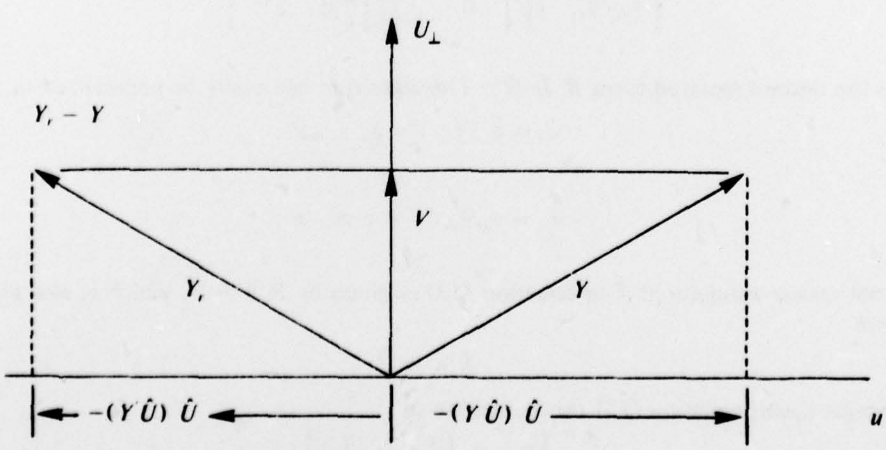

Fig. 1 — Geometry of the Householder algorithm

where β is defined as $\beta = 2/U'\,DU$. The matrix T is an elementary Householder transform with properties $T = T'$ and $T^2 = I$, and it can be shown to triangulate a matrix.

To obtain the desired results required in the previous section, the dot products of equation (23) are modified by a diagonal matrix D to yield

$$Y_r = Y - 2\frac{(Y'DU)}{(U'DU)}\,U = (I - \beta UU'D)\,Y = TY. \tag{24}$$

Four properties are given and proved. Property 1 is

$$T' = T \ (T \text{ is symmetric}).$$

Since, from equation (35),

$$\begin{aligned} T' &= (I - \beta UU'D)' \\ &= (I - \beta D'UU') \\ &= (I - \beta DUU'), \end{aligned}$$

the proof that $T' = T$ requires showing that $DUU' = UU'D$. If $DUU' = UU'D$, then

$$U'(DUU')\,U = U'(UU'D)\,U$$

or

$$(U'DU)\,(U'U) = (U'U)\,(U'DU),$$

which is an identity. QED.

Property 2 is

$$T'\,DT = D.$$

Since

$$\begin{aligned} T'DT &= (I - \beta UU'D)'\,D\,(I - \beta UU'D) \\ &= (I - \beta DUU')\,D\,(I - \beta UU'D) \\ &= (D - \beta DUU'D)\,(I - \beta UU'D) \\ &= D - 2\beta DUU'D + \beta^2 DUU'DUU'D \\ &= D - 2\beta DU\left[U'D - (\beta/2)\,U'DUU'D\right], \end{aligned}$$

the proof requires showing that

$$U'D - (\beta/2)\,U'DUU'D = 0$$

or that

$$U'D - \frac{2}{2(U'DU)}\,U'DUU'D = 0$$

or that

$$U'DUU'D - U'DUU'D = 0,$$

which is an identity. QED.

To express properties 3 and 4, the following notation is introduced:

$$\sigma = \sqrt{\frac{Y'DY}{d_1}}, \quad \text{where } D = \begin{bmatrix} d_1 & 0 & 0 & 0 \\ 0 & d_2 & 0 & 0 \\ 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & d_n \end{bmatrix}$$

and

$$U = \begin{bmatrix} u_1 \\ u_2 \\ \dots \\ \mu_n \end{bmatrix}$$

in which

$$u_1 = y_1 + \sigma,$$
$$u_2 = y_2,$$
$$u_3 = y_3,$$

and

$$u_N = y_N.$$

Property 3 is

$$\beta = \frac{2}{U'DU} = \frac{1}{d_1 \sigma u_1},$$

which is proven by writing

$$\begin{aligned} \frac{2}{U'DU} &= \frac{2}{(y_1+\sigma)^2 d_1 + y_2^2 d_2 + \dots + y_n^2 d_n} \\ &= \frac{2}{2y_1\sigma d_1 + \sigma^2 d_1 + (y_1^2 d_1 + y_2^2 d_2 + \dots + y_n^2 d_n)} \\ &= \frac{2}{2y_1\sigma d_1 + \sigma^2 d_1 + \sigma^2 d_1} \\ &= \frac{1}{d_1\sigma(y_1+\sigma)} = \frac{1}{d_1 \sigma u_1}. \end{aligned}$$

QED.

Property 4 is

$$Y_r = TY = \begin{bmatrix} y_{r1} \\ y_{r2} \\ \dots \\ y_{rn} \end{bmatrix} = \begin{bmatrix} -\sigma \\ 0 \\ \dots \\ 0 \end{bmatrix}$$

The proof is as follows. From equation (35)

$$
\begin{aligned}
TY &= Y - 2\frac{(Y'DU)}{U'DU}\,U \\
&= Y - \frac{\left[y_1(y_1+\sigma)d_1 + y_2^2 d_2 + \ldots + y_N^2 d_n\right]U}{d_1\sigma u_1} \\
&= Y - \frac{(y_1\sigma d_1 + \sigma^2 d_1)\,U}{d_1\sigma u_1} \\
&= Y - \frac{(y_1+\sigma)}{u_1}\,U \\
&= Y - U \\
&= \begin{bmatrix} y_1 \\ y_2 \\ \ldots \\ y_N \end{bmatrix} - \begin{bmatrix} y_1+\sigma \\ y_2 \\ \ldots \\ y_N \end{bmatrix} = \begin{bmatrix} -\sigma \\ 0 \\ \ldots \\ 0 \end{bmatrix}
\end{aligned}
$$

QED. We will use

$$TY = -Y + 2\frac{(Y'DU)}{U'DU}\,U$$

such that

$$TY = \begin{bmatrix} +\sigma \\ 0 \\ \ldots \\ 0 \end{bmatrix}$$

so as to have the convenience of postive diagonal elements in the resulting triangularized matrix. Properties 3 and 4 can be used to triangulate a matrix by applying them on successive columns of the matrix.

The following example illustrates the use of the algorithm:

$$T_1 \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{bmatrix} = \begin{bmatrix} \sigma_1 & b_{12} & b_{13} \\ 0 & b_{22} & b_{23} \\ 0 & b_{32} & b_{33} \\ 0 & b_{42} & b_{43} \end{bmatrix}$$

where

$$U = \begin{bmatrix} a_{11} + \sigma_1 \\ a_{21} \\ a_{31} \\ a_{41} \end{bmatrix}$$

$$\sigma_1 = \operatorname{sgn} a_{11}\sqrt{(a_{11}^2 d_1^2 + a_{21}^2 d_2 + a_{31}^2 d_3 + a_{41}^2 d_4)/d_1},$$

and

$$b_{ij} = -a_{ij} + \beta\gamma_{1j}u_i, \quad j = 1, \ldots, 3 \text{ and } i = 1, \ldots 4,$$

in which

$$\beta = \frac{1}{d_1\sigma u_1}$$

and

$$\gamma_{1i} = a_{1j}u_1 + a_{2j}u_2 + a_{2u}u_2 + a_{3j}u_3 + a_{4j}u_4.$$

The process is repeated for each successive submatrix. The next step is

$$\begin{bmatrix} 1 & 0 \\ 0 & T_2 \end{bmatrix} \begin{bmatrix} \sigma_1 & b_{12} & b_{13} \\ 0 & b_{22} & b_{23} \\ 0 & b_{32} & b_{33} \\ 0 & b_{42} & b_{43} \end{bmatrix} = \begin{bmatrix} \sigma_1 & b_{12} & b_{13} \\ 0 & \sigma_2 & c_{23} \\ 0 & 0 & c_{33} \\ 0 & 0 & c_{43} \end{bmatrix}$$

where

$$U = \begin{bmatrix} b_{22} + \sigma \\ b_{32} \\ b_{42} \end{bmatrix}$$

$$\sigma_2 = \operatorname{sgn}(b_{22}) \sqrt{(b_{22}^2 d_2 + b_{32}^2 d_3 + b_{42}^2 d_4)/d_2},$$

and

$$c_{ij} = -b_{ij} + \beta\gamma_{2j}u_{j-1}, \quad j = 2, 3 \text{ and } i = 2, \ldots 4,$$

in which

$$\beta = \frac{1}{d_2\sigma u_1}$$

and

$$\gamma_{2j} = b_{2j}u_1 + b_{3j}u_2 + b_{4j}u_3.$$

In summary, it was shown that the modified Householder algorithm meets the requirements of the modified SRIF filter ($T'D\ T = D$, with T triangulating the matrix). A simple means of obtaining the prediction portion of the SRIF filter under an important case is next considered.

PREDICTION PROCESS

The smoothing portion of the Kalman filter using a modified-SRIF-filter implementation updates the factorization of the smoothed covariance and the transformed best estimate. It is desirable to update the prediction process in a commensurable form. Only an important special case is considered: The process noise $W(k)$ is assumed to be zero, and the state transition matrix is assumed to be in the upper triangular form. Equation (4) updating the predicted covariance then becomes

$$\hat{P} = \Phi\, P\, \Phi', \tag{25}$$

where the noise $W(k)$ is removed and the sample k has been dropped for notational convenience. The inverse of (25) is taken, yielding

$$\hat{P}^{-1} = (\Phi')^{-1}\,\tilde{P}^{-1}\,\Phi^{-1}. \tag{26}$$

The covariances are replaced with their factorizations

$$\hat{R}\,\hat{D}\,\hat{R}' = (\Phi')^{-1}\,\tilde{R}\,\tilde{D}\,\tilde{R}'\,\Phi^{-1},$$

which can be rewritten as

$$\hat{R}\,\hat{D}\,\hat{R} = (\Phi')^{-1}\,\tilde{R}\,\tilde{D}(\Phi^{-1}\tilde{R})'.$$

Since $(\Phi')^{-1}\,R$ is of the lower triangular form, then

$$\hat{R} = (\Phi')^{-1}\,\tilde{R} \tag{27a}$$

and

$$\hat{D} = \tilde{D}, \tag{27b}$$

which shows the simple form of updating the factors of the prediction covariance.

The predicted state given by

$$\hat{X} = \Phi\,\tilde{X}$$

from equation (7) is transformed by

$$(\hat{R}')^{-1}\,\hat{Z} = \Phi(\tilde{R}')^{-1}\,\tilde{Z},$$

where $\hat{X} = (\hat{R}')^{-1}\,\hat{Z}$ and $\tilde{X} = (\tilde{R}')^{-1}\,\tilde{Z}$. Solving for $\hat{Z}$ yields

$$\hat{Z} = (\hat{R}')\,\Phi(\tilde{R}')^{-1}\,\tilde{Z}. \tag{28}$$

Substituting $\hat{R}$ from (27a) into (28) yields

$$\hat{Z} = \tilde{Z}.$$

The transformed smoothed and predicted states are seen to be identical.

Sometimes it is desirable to implement a fading-memory filter by making the smoothed covariance larger. This is accomplished by rewriting equation (26) as

$$\hat{P}^{-1} = (\Phi')^{-1}\,a\tilde{P}^{-1}\,\Phi^{-1}.$$

The parameter is a scalar representing a time fading by

$$a = e^{-t/\tau},$$

where τ is the time constant and t is time. Equation (27b) is modified, and equations (27a) and (27b) now are

$$\hat{R} = (\Phi')^{-1}\,\tilde{R}$$

and

$$\hat{D} = \sqrt{a}\,\tilde{D},$$

and these equations now remain the same under the fading-memory condition.

A functional flow of the filter using the special-case prediction is shown in Fig. 2. The measurement and measurement matrix are first transformed using the factorization of the measurement covariance matrix. For zero process noise and upper-triangular-form state transition matrices, the prediction process is given simply. The transformed predicted and smoothed states are the same, and the covariances are related by a simple transform. The smoothing on

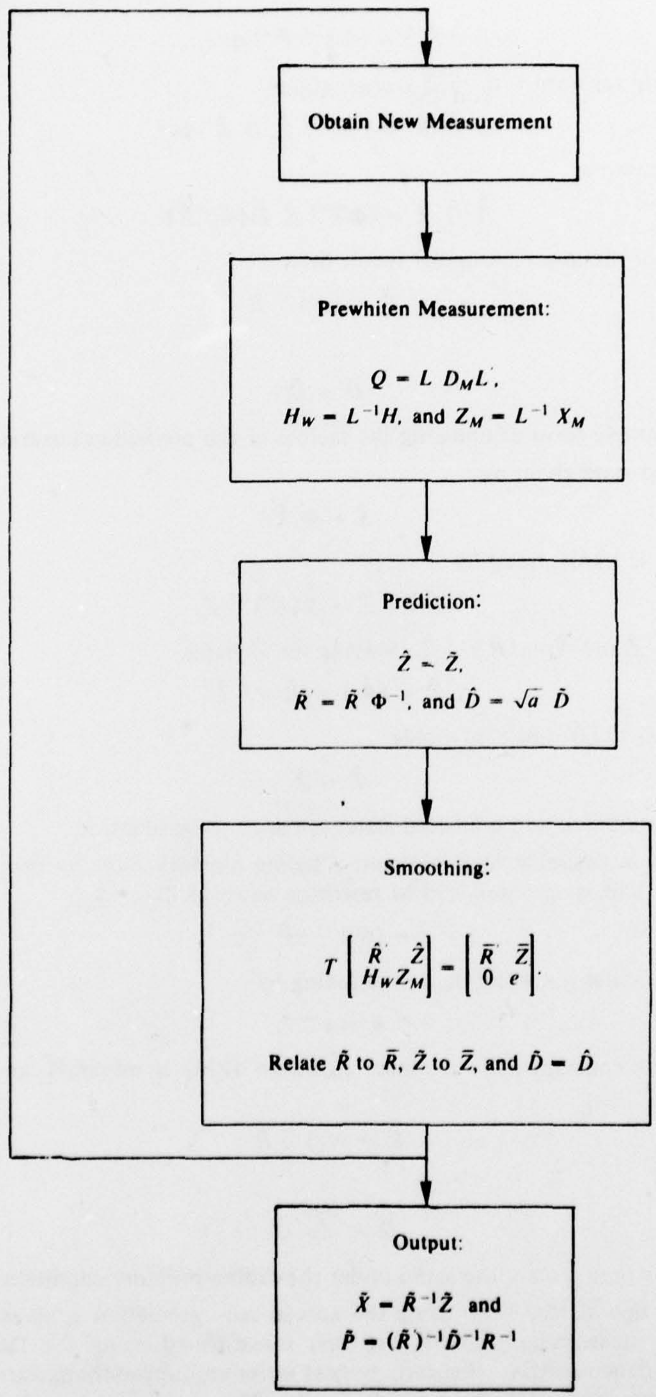

Fig. 2 — Functional flow of the modified SRIF filter using LDL' factorization and a special-case prediction

the filter is given by the modified Householder transform previously described, and the terms are rearranged to obtain a more desirable form. This completes the filter operation except for forming the smoothed estimate and the covariance if desired from the transformed state and the covariance factors.

SUMMARY

The SRIF filter is a numerical means of solving the Kalman-filter equations. In this report the SRIF filter was modified to accommodate LDL factorizations of the covariances rather that the usual LL factorizations. This factorization required that the Householder algorithm be modified and required a small manipulation of the results after transforming. Under a special case the prediction process was shown to be quite simple using the LDL′ factorization.

The reason for investigating the SRIF fiter using LDL′ factorizations was to try to eliminate the square-root operations found in the SRIF filter. Although the square-root operations using LDL′ factorization now do not appear in the prewhitening process, they still are necessary in the modified Householder algorithm. Consequently the results are not as strong as was first hoped for. An interesting result does occur in the use of a fading memory: Only the diagonal elements D of the LDL′ factorization need be weighted rather than the entire covariance matrix.

REFERENCES

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